Dispersion Characteristics of Square Pulse with Finite Rise Time in Single, Tapered, and Coupled Microstrip Lines

Protap Pramanick, *Senior Member, IEEE*, and Raafat R. Mansour, *Senior Member, IEEE*

Abstract —**The distortion of an electrical pulse, with finite rise time (quadratic-linear-quadratic transition) caused by dispersion as it propagates along a uniform microstrip, a tapered microstrip and a coupled pair of microstrips is investigated. Closed form analysis equations for single and coupled microstrips have been used to find the frequency dependent phase velocities. Results have been presented for two different taper profiles (exponential and triangular distributions). It is concluded that the optimization of the taper profile will provide the least pulse distortion.**

I. Introduction

MICROSTRIP line finds extensive use in modern fast computers, phased array antennas, filters etc. The knowledge of time domain analysis reveals many useful facts regarding the frequency domain behavior of such microstrip components [1]. In addition, ultra-fast switching speeds and decreasing circuit dimensions of MMIC's cannot ignore the dispersive role of phase velocity in microstrip of various forms used in MMIC's.

Dispersion of dc and RF pulses in waveguides and other transmission lines have been investigated [2]–[4]. Veghte and Balanis [5] studied the dispersion of ideal square, Gaussian and modulated Gaussian pulse in uniform microstrip line. Leung and Balanis [6] extended the analysis to lossy microstrip lines. Full-wave spectral domain analysis of the phase velocity of microstrip was used by Leung and Balanis [7] in pulse distortion analysis of shielded and open microstrips. Gilb and Balanis [8] reported Gaussian pulse distortion analysis in more generalized multi-layered coupled microstrip lines. However, the distortion of a nonideal square pulse, i.e., a square pulse with finite rise time in a nonuniform or coupled pair of microstrips has not been analyzed. Since in a real situation all square pulses are nonideal, such an analysis is required.

A pulse consists of an infinite number of pure sinusoids with increasing frequencies and decreasing amplitudes. Since in a microstrip, the phase velocity depends upon frequency, different components of a square pulse propagates at different phase velocities. Consequently, a distorted pulse shape results at the receiving end of the microstrip line.

Tapered and coupled lines form the basic building blocks of many microstrip components. This paper presents the analysis and the results of the distortion of a nonideal square pulse, shown in Fig. 1, as it propagates along a uniform, an exponentially tapered, a triangularly tapered and a coupled pair of microstrip lines.

Manuscript received April 9, 1991; revised July 29, 1991.

The authors are with COM DEV Ltd., 155 Shelton Drive, Cambridge, ON, Canada N1R 7H6.

IEEE Log Number 9103932.

II. Theory

A. Tapered Line

Let us consider a tapered transmission line shown in Fig. 2. The transfer function of the transmission line can be written as [9]

$$T(\omega)=\left[1-\frac{1}{4}\left[\ln\left(\frac{Z_L}{Z_o}\right)F(\theta(\omega))\right]^2\right]^{1/2}e^{-j\theta(\omega)} \quad (1)$$

where $F(\theta(\omega))$ depends on the type of the profile chosen. As an example, for an exponential line [9]

$$F(\theta(\omega))=\frac{\sin(\theta(\omega))}{\theta(\omega)}. \quad (2)$$

For a triangular distribution of the impedance profile [9],

$$F(\theta(\omega))=\left[\frac{\sin(\theta(\omega)/2)}{\theta(\omega)/2}\right]^2 \quad (3)$$

where

$$\theta(\omega)=\int_0^L\beta(\omega,z)\,dz. \quad (4)$$

$\beta(\omega,z)$ is the propagation constant profile. For a nonideal square pulse with linear-square-linear transition, as shown in Fig. 1, the time domain representation is given by

$$V(t,z=0)=\begin{Bmatrix} 1 & 0<t<T_1 \\ 1-a(t-T_1)^2 & T_1<t<T_1+q\tau_1 \\ bt+e & T_1+q\tau_1<t<\tau-q\tau_1 \\ a(t-\tau)^2 & \tau-q\tau_1<t<\tau \\ 0 & t>\tau \end{Bmatrix} \quad (5)$$

0018-9480/91$01.00

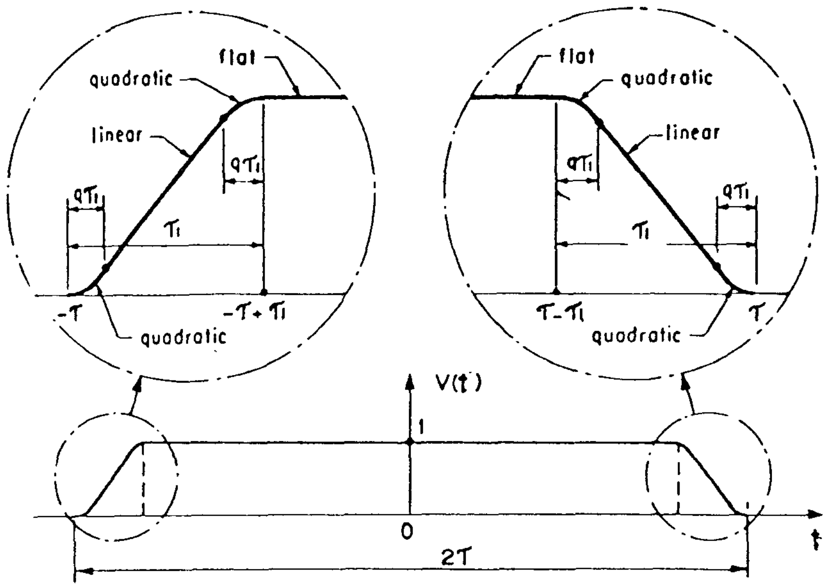


Fig. 1. Square pulse with finite rise time.

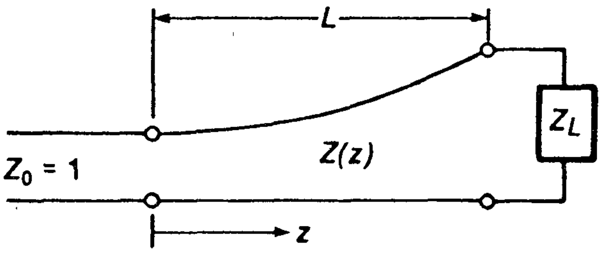


Fig. 2. Tapered transmission line.

where $T_1 = \tau - \tau_1$, $\tau_1 < \tau$, $0 < q < 0.5$ and

$$a = \frac{-1}{2a(1-q)\tau_1^2}, \quad b = \frac{-1}{(1-q)\tau_1}, \quad e = \frac{2\tau - q\tau_1}{2(1-q)\tau_1}.$$

The Fourier transform of the pulse is given by [10]

$$V(\omega, z=0) = \frac{8}{q(1-q)\tau_1^2} \times \frac{\sin\left[\left(\tau - \frac{\tau_1}{2}\right)\omega\right] \sin\left[\frac{q\tau_1\omega}{2}\right] \sin\left[\frac{(1-q)\tau_1\omega}{2}\right]}{\omega^3}. \quad (6)$$

Fig. 3 shows that the amplitude of a non-ideal pulse decays faster than the ideal square pulse in the frequency domain. At $z = L$ along the tapered line, the pulse in the frequency domain becomes

$$V(\omega, z=L) = V(\omega, z=0)|T(\omega)|e^{-j\theta(\omega)}. \quad (7)$$

Taking the inverse transform of (7) gives the time domain representation of the pulse at $z = L$ which can be written as

$$V(t, L) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(\omega, z=0)|T(\omega)|e^{j(\omega t - \theta(\omega))}\, d\omega. \quad (8)$$

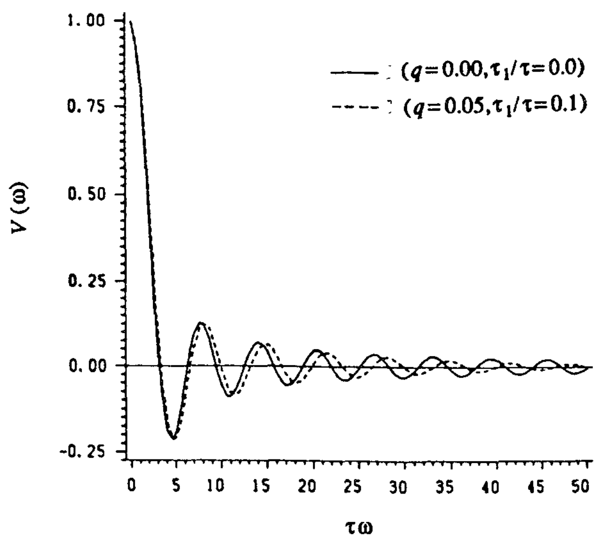


Fig. 3. Frequency domain representation of ideal and nonideal square pulses.

In (8), for each value of ω, $\theta(\omega)$ is obtained by numerically evaluating the integral in (4). The propagation constant profile is given by

$$\beta(\omega, z) = \frac{\omega}{c}\sqrt{\epsilon_{\text{eff}}(\omega, z)} \quad (9)$$

Where c is the velocity of light in free space and the frequency dependent effective dielectric constant profile is given by [5], [11]

$$\epsilon_{\text{eff}}(\omega, z) = \epsilon_r - \frac{\epsilon_r - \epsilon_{\text{eff}}(0, z)}{1 + \frac{\epsilon_{\text{eff}}(0, z)}{\epsilon_r}\left(\frac{\omega}{\omega_p}\right)^2} \quad (10)$$

where $\omega_p = \frac{Z_o(0, z)\pi}{\mu_o h}$,

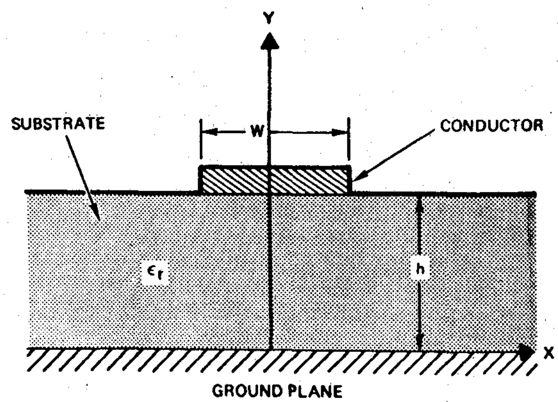


Fig. 4. Microstrip line.

$Z_o(0, z)$ is the zero frequency characteristic impedance of the tapered line at a point z. μ_o is the free-space permeability and h is the substrate thickness of the microstrip shown in Fig. 4:

$$\epsilon_{\text{eff}}(0,z)=\frac{\epsilon_r+1}{2}\cdot\left[1+\frac{30}{Z_o(0,z)}\sqrt{\frac{2}{\epsilon_r+1}}\,\frac{\epsilon_r-1}{\epsilon_r+1}\left[\ln\frac{\pi}{2}+\frac{1}{\epsilon_r}\ln\left(\frac{4}{\pi}\right)\right]\right]^2 \quad (11)$$

for $Z_o(0, z) \geqslant 63-2\epsilon_r$

and

$$\epsilon_{\text{eff}}(0,z)=\epsilon_r[0.96+\epsilon_r(0.109-0.004\epsilon_r)]\cdot[\log(10+Z_o(0,z))]^{-1} \quad (12)$$

for $Z_o(0, z) \leqslant 63-2\epsilon_r$. In (8), the limits of integration are $-\infty$ and $+\infty$. However, one need not have to consider the entire range, because beyond a certain frequency limit, the contribution to the integral is negligible. This is more true for a slow rising wider pulse. Therefore (8) can be written as

$$V(t,L)=\frac{1}{2\pi}\int_{-\omega_L}^{\omega_L} V(\omega,z=0)|T(\omega)|e^{j(\omega t-\theta(\omega))}\,d\omega. \quad (13)$$

Different values of ω_L are tried to obtain the convergence of the integration. Since we are concerned with the real part only, (13) can be written as

$$V(t,L)=\frac{1}{2\pi}\int_{-\omega_L}^{\omega_L} V(\omega,z=0)|T(\omega)|\cdot\cos(\omega t-\theta(\omega))\,d\omega. \quad (14)$$

Equation (14) is numerically evaluated using a suitable computer program. The limit of integration ω_L is chosen on the basis of the criterion described in [5].

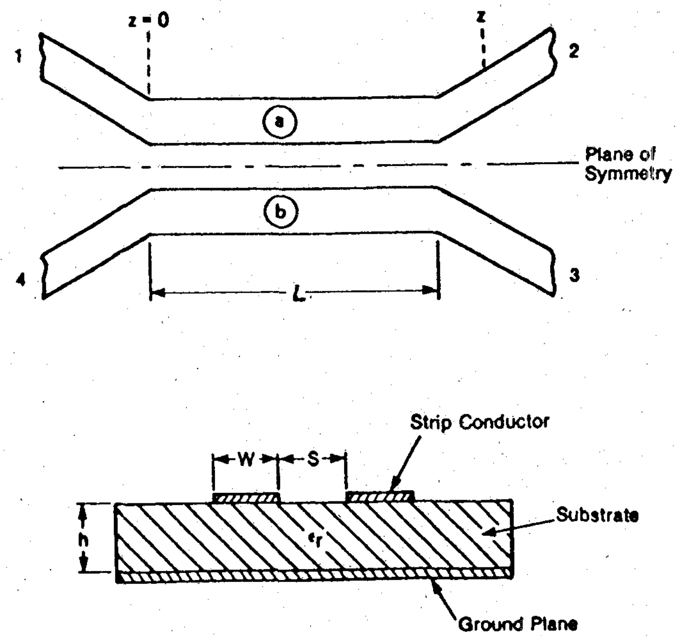


Fig. 5. Coupled line.

B. Pair of Uniform Coupled Lines

The best way to analyze the response of the coupled pair of transmission lines shown in Fig. 5, is to split the input signal into even- and odd-modes. In the even-mode two in phase signals of half the amplitude of the original signal are fed to both the microstrips while in the odd-mode, the same two signals are fed but with a 180° phase opposition to each other. The effective dielectric constant for each mode can be easily calculated using Kirschning and Jansen's [12] closed-form equations. The response to lines 1 and 2 for signal in line 1 only are given by

$$v_1(t,L)=\frac{1}{2}[v_e(t,L)+v_o(t,L)] \quad (15a)$$

$$v_2(t,L)=\frac{1}{2}[v_e(t,L)-v_o(t,L)], \quad (15b)$$

where $v_e(t, L)$ and $v_o(t, L)$ are even- and odd-mode responses of line 1 to the input signal

$$v_e(t,L)=\frac{1}{2\pi}\int_{-\omega_L}^{\omega_L} V(\omega,z=0)e^{j(\omega t-\beta_e(\omega)L)}\,d\omega \quad (16a)$$

$$v_o(t,L)=\frac{1}{2\pi}\int_{-\omega_L}^{\omega_L} V(\omega,z=0)e^{j(\omega t-\beta_o(\omega)L)}\,d\omega \quad (16b)$$

$$\beta_e=\frac{\omega}{c}\sqrt{\epsilon_e(\omega)} \quad (17a)$$

$$\beta_o=\frac{\omega}{c}\sqrt{\epsilon_o(\omega)} \quad (17b)$$

$\epsilon_e(\omega)$ and $\epsilon_o(\omega)$ are the even-mode and the odd-mode effective dielectric constants, respectively of the microstrips. Following the arguments in the previous section, (15a) and (15b) can be written as

$$v_1(t,L) = \frac{+1}{2\pi}\int_{-\omega_L}^{+\omega_L} V(\omega, z=0) \cdot \cos\left[\frac{\omega L}{c}\frac{\sqrt{\epsilon_e(\omega)}-\sqrt{\epsilon_o(\omega)}}{2}\right] \cdot \cos\left[\omega t - \frac{\omega L}{c}\frac{\sqrt{\epsilon_e(\omega)}+\sqrt{\epsilon_0(\omega)}}{2}\right] d\omega \quad (18a)$$

$$v_2(t,L) = \frac{-1}{2\pi}\int_{-\omega_L}^{+\omega_L} V(\omega, z=0) \cdot \sin\left[\frac{\omega L}{c}\frac{\sqrt{\epsilon_e(\omega)}-\sqrt{\epsilon_o(\omega)}}{2}\right] \cdot \sin\left[\omega t - \frac{\omega L}{c}\frac{\sqrt{\epsilon_e(\omega)}+\sqrt{\epsilon_0(\omega)}}{2}\right] d\omega. \quad (18b)$$

Equations (18a) and (18b) can be numerically evaluated using a suitable computer program. As mentioned above $\epsilon_e(\omega)$ and $\epsilon_o(\omega)$ are obtained from Kirschning and Jansen's [12] equations.

III. Numerical Results

A. Uniform and Tapered Microstrip

The dispersions of a nonideal square pulse in a one inch long straight and tapered microstrip for various impedance ratios are shown in Fig. 6(a) and (b), for exponential [9] and triangular [9] profiles, respectively. We observe from the figures that the effect of the ratio Z_L/Z_o on the delay of the pulse is more in the exponential profile than on the triangular profile. Fig. 7(a) and (b) show that for $Z_L/Z_o = 2$ the difference in delay is almost negligible. However, it increases as Z_L/Z_o increases. In all the above cases we find that as the ratio Z_L/Z_o increases, the amplitude of the transmitted pulse reduces. This is expected because for the same length of the taper, higher Z_L/Z_o produces larger passband ripple in frequency response of the taper giving rise to larger mismatch, consequently larger reflected signal and smaller transmitted signal. The amplitude of the reflected signal can be calculated using the following equation:

$$V(t,0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(\omega, z=0)|R(\omega)|e^{j(\omega t-\theta(\omega))}d\omega$$

$$R(\omega) = \frac{1}{2}\ln\left(\frac{Z_L}{Z_o}\right)F(\theta(\omega))e^{-j\theta(\omega)}.$$

Fig. 8(a) and (b) show the reflected pulses for exponential and triangular distributions, respectively. We observe that for $Z_L = 2Z_o$, the difference in pulse distortions between exponential and triangular distributions is negligible. However, the difference is obvious for $Z_L = 3Z_o$. From the computed results we may conclude that a Chebyshev

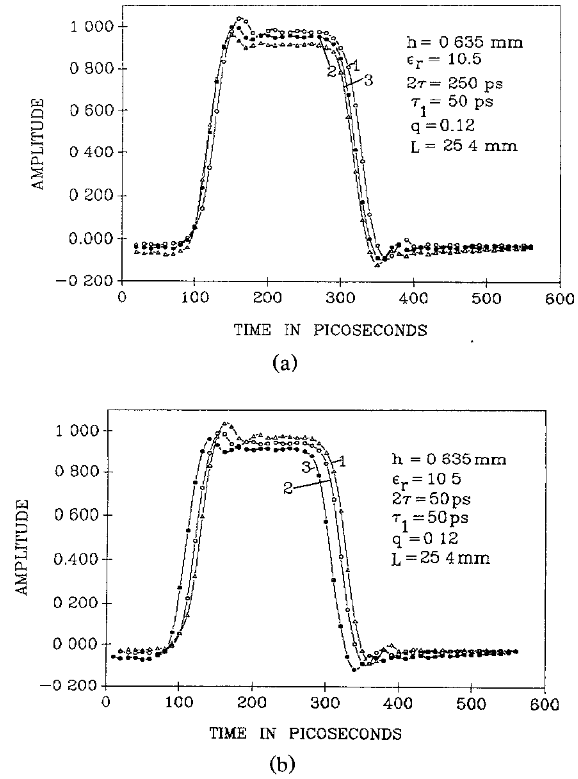


Fig. 6. (a) Distorted pulse for an exponential taper. 1) $Z_L/Z_o = 1$. 2) $Z_L/Z_o = 2$. 3) $Z_L/Z_o = 3$. (b) Distorted pulse for a triangular taper. 1) $Z_L/Z_o = 1$. 2) $Z_L/Z_o = 2$. 3) $Z_L/Z_o = 3$.

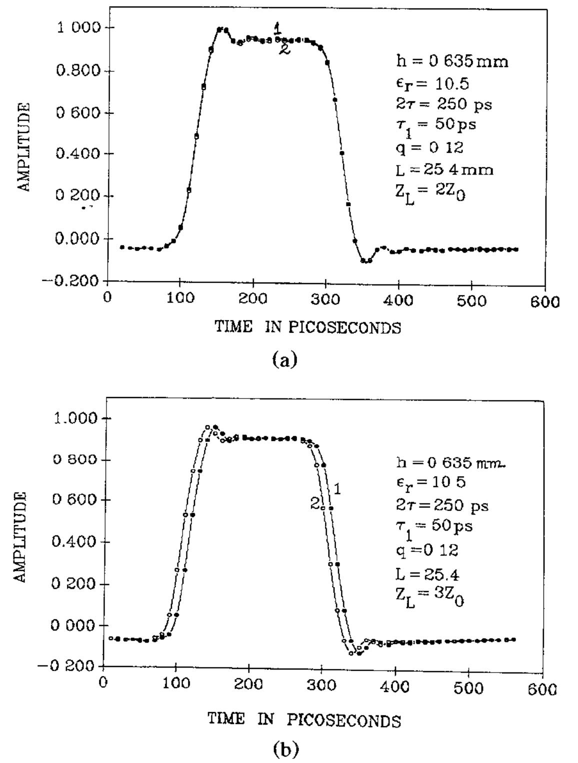
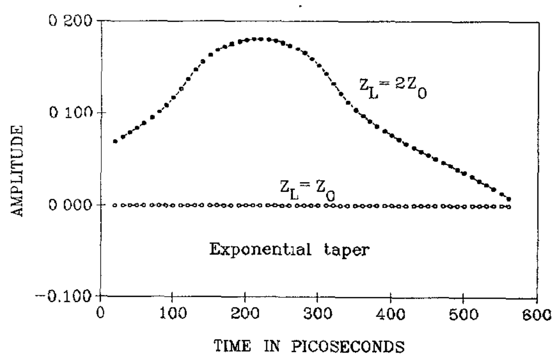
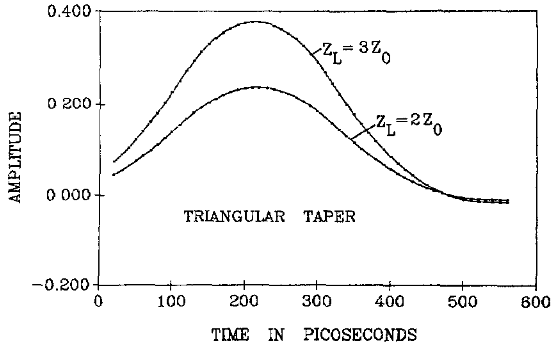


Fig. 7. (a) Comparison of dispersed pulse for different taper profiles. 1) Exponential. 2) Triangular, $Z_L = 2Z_o$ (b) Comparison of dispersed pulse for different taper profiles. 1) Exponential. 2) Triangular. $Z_L = 3Z_o$



(a)



(b)

Fig. 8. (a) Dispersion of reflected pulse in exponential taper. (b) Dispersion of reflected pulse in triangular taper.

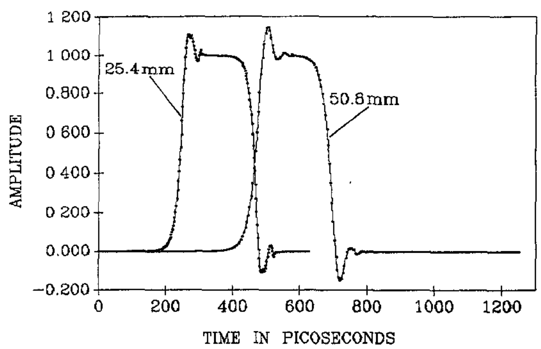
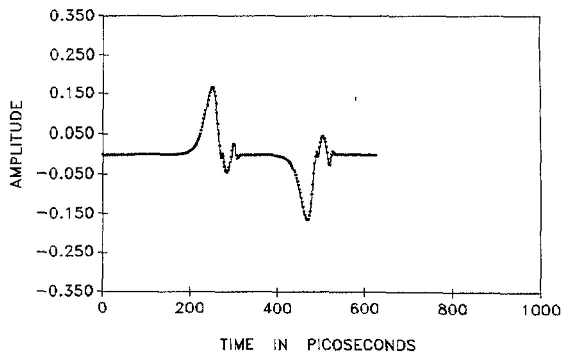


Fig. 9. Dispersed pulse in coupled microstrips: for $L = 25.4$ mm, and $L = 50.8$ mm, $h = 0.635$ mm, $\epsilon_R = 10.2$, $S = 2$ mm, $W = 0.635$ mm, $q = 0.005$ (SUM).

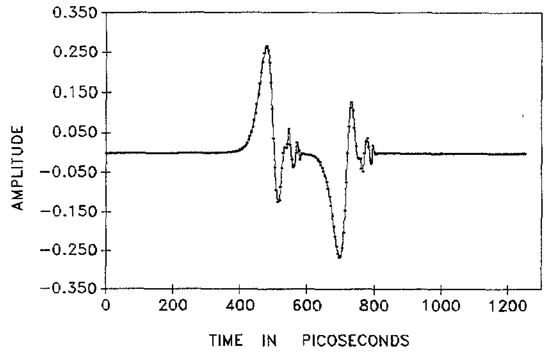
taper will offer the minimum pulse distorsion due to its minimum passband ripples and the shortest length.

B. Coupled Uniform Microstrips

Figs. 9 and 10 show the pulse response of a coupled pair of microstrip lines, on a 25 mil substrate of dielectric constant 10.5, to a nonideal square pulse. The even and the odd-mode pairs of pulses add constructively on the signal line and destructively on the sense line. As the signals start out, the even and the odd pairs separate from each other due to their different phase velocities. What have been shown in Figs. 9 and 10 are the difference of the separated signals in the sense line and the sum in the signal line for two different line lengths. The total pulse



(a)



(b)

Fig. 10. (a) Dispersed pulse in coupled microstrips: for $L = 25.4$ mm, $h = 0.635$ mm, $\epsilon_r = 10.2$, $S = 2$ mm, $W = 0.635$ mm, $q = 0.005$ (DIFFERENCE). (b) Dispersed pulse in coupled microstrips: for $L = 50.8$ mm, $h = 0.635$ mm, $\epsilon_r = 10.2$, $S = 2$ mm, $W = 0.635$ mm, $q = 0.005$ (DIFFERENCE).

spread, in time due to the difference between the odd and the even mode phase velocities at a distance L, can be written as [8]

$$\Delta t = \frac{L}{c}\left[\sqrt{\epsilon_e(\omega)} - \sqrt{\epsilon_e(\omega)}\right]$$

where c is the speed of light in free-space.

IV. Conclusion

This paper has analyzed the distortion, due to dispersion, of a nonideal square pulse (quadratic-linear-quadratic transition) in uniform, tapered and coupled microstrip lines. It has been observed from the analysis that a faster impedance variation along the taper gives rise to smaller amplitude of the transmitted pulse and larger amplitude of the reflected pulse. Detailed comparisons have been made between the effects of exponential and triangular taper profiles, and different impedance ratios matched by the tapers. Numerical results have also been presented for coupled microstrip lines.

References

[1] X. Zhang, J. Fang, K. K. Mei, and Y. Tiu, "Calculation of dispersion characteristics of microstrips by time domain finite difference method," *IEEE Trans. Microwave Theory Tech.*, vol. 36, no. 2, pp. 263–267, Feb. 1988.

[2] R. S. Elliot, "Pulse waveform degradation due to dispersion in waveguide," *IRE Trans. Microwave Theory Tech.*, vol. 5, no. 10, pp. 254–257, Oct. 1957.

[3] C. M. Knopp, "Pulsed electromagnetic wave propagation in dispersive media," *IEEE Trans. Antennas Propagat.*, vol. AP-12, no. 7, pp. 494–496, 1964.
[4] M. P. Forrer, "Analysis of millisecond RF pulse transmission," *Proc. IRE*, vol. 46, no. 6, pp. 1830–1835, Nov. 1958.
[5] R. L. Veghte and C. A. Balanes, "Dispersion of transient signals in microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 34, no. 12, pp. 1427–1432, Dec. 1986.
[6] T. Leung and C. A. Balanis, "Attenuation distortion of transient signals in microstrip," IEEE Trans, vol. 36, no. 4, pp. 765–769, Apr. 1988.
[7] ——, , "Pulse dispersion in open and shielded microstrips using the spectral-domain method," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1223–1226, July 1988.
[8] J. P. Gilb and G. A. Balanis, "Pulse distortion on multilayered coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 10, pp. 1620–1628, Oct. 1989.
[9] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw Hill, 1966.
[10] R. MacPhie and R. R. Mansour, "Cross correlation antenna system with modified principal solution spatial frequency transfer function," *IEEE Trans. Antennas Propagat.*, vol. AP-33, no. 3, pp. 312–320, Mar. 1985.
[11] P. Pramanick and P. Bhartia, "An accurate description of dispersion in microstrip," *Microwave J.*, pp. 89–92, Dec. 1983.
[12] M. Kirschning and R. Jansen, "Accurate wide range dynamic equations for the frequency dependent characteristics of parallel coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, no. 1, pp. 83–90, Jan. 1984.

Protap Pramanick (S'83–M'84–SM'88) received the B.Tech (Hons) degree in electronics and communication engineering from the Indian Institute of Technology in 1977 and the Ph.D degree in Electrical Engineering from the Indian Institute of Technology, Kanpur, India, in 1982.

In September 1982 he joined the Department of Electrical Engineering, University of Ottawa, Ottawa, ON, Canada, as a Post Doctoral Fellow. In December 1984, he joined the Canadian Marconi Company, Ottawa, as a member of the Microwave Group, Avionics Division. Since April 1986, he has been with the Satellite Communication Department of COM DEV Limited in Cambridge, Ontario.

Dr. Pramanick has published extensively in the field of microwave and millimeter-wave MIC's. He also holds a number of patents on filters for cellular radio. He co-edited the book *E-Plane Integrated Circuits* (Artech House, 1987), contributed to the books *Topics in Millimeter-Wave Technology* (Academic Press, 1988), *Microwave Solid State Circuit Design* (Wiley Interscience, 1988) and *Handbook of Microwaves* (Academic Press, to appear). He is on the editorial review board of IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.

Raafat R. Mansour (S'84–M'86–SM'90) was born in Cairo, Egypt, on March 31, 1955. He received the B.Sc (with honors) and M.Sc degrees from Ain Shams University, Cairo, in 1977 and 1981, respectively, and the Ph.D. degree from the University of Waterloo, Ont., Canada, in 1986, all in electrical engineering.

He was a research fellow at the Laboratoire d'Electromagnetisme, Institut National Polytechnique, Grenoble, France, in 1981. From 1983 to 1986 he was a Research and Teaching Assistant with the Department of Electrical Engineering, University of Waterloo. Since then he has been with COM DEV Ltd., Cambridge, ON, Canada, where he is currently a Senior Member of the Technical Staff. His present research interests are in the analysis and design of microwave and millimeter-wave integrated circuits.

Dr. Mansour received the 1987 University of Waterloo Engineering award for outstanding achievement. He was also awarded a Natural Sciences and Engineering Research Council of Canada NSERC Fellowship in 1987. He is on the Editorial Board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.